# Technical Notes

*TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).*

## Improved Thermal Stress Determination by Finite Element Methods

I. U. OJALVO*
*Grumman Aerospace Corporation, Bethpage, N.Y.*

### Introduction

A MAJOR consideration for application of the finite element method is in the selection of an adequate structural idealization. Various factors such as judgement, experience, desired accuracy, and efficiency have dictated the structural grids actually employed. With the expansion of finite element methods into the thermal stress area, consideration has also been given to the importance of temperature gradients upon mesh-size selection.

The added complexity associated with generating idealizations which are adequate for determining thermal stresses arises from the fact that stresses are generally highest when temperature gradients are most severe. Thus, to obtain accuracy levels comparable to those for unheated structures, it has been necessary to work with finer grid sizes. A related problem has been that the thermal stress analysis of a single component has required the use of multiple idealizations to accommodate various time slices, each associated with a different transient temperature state.

The purpose of this Note is to indicate that a structural mesh refinement can be avoided in most thermal stress problems by taking advantage of a basic fact. This fact, which will be demonstrated, is that strains in heated structures idealized by conventional components are generally less sensitive to spatially distributed temperature variations than are their corresponding stresses. Hence, it would appear that an efficient stress calculation procedure would be to employ only the strains from a coarse-mesh finite element solution. These would then be combined with a finer-mesh temperature distribution and the stress-strain-temperature constitutive equations, to obtain fine-mesh stress results. A supplementary advantage of this approach is that a single structural grid will suffice for a component undergoing a variety of transient temperature states.

### Technical Approach

Assume that a structural element with arbitrary instantaneous temperature distribution $T$ is statically supported such that its stiffness matrix $[K]$ is nonsingular. Its nodal deflections $\{\delta\}$ may be obtained from the equation[1]

$$[K]\{\delta\} = \{P_{\text{mech}}\} + \{P_T\} \tag{1}$$

where

$$\{P_T\} = \int_V T(x,y,z)[B]^T[D]\{\alpha\}\,dV \tag{2}$$

The matrices $[K]$, $[B]$, and $[D]$ are related to the stress and strain vectors, $\{\sigma\}$ and $\{\varepsilon\}$, respectively, through the relationships

$$[K] = \int_V [B]^T[D][B]\,dV \tag{3}$$

$$\{\varepsilon\} = [B]\{\delta\} \tag{4}$$

$$\{\sigma\} = [D](\{\varepsilon\} - T\{\alpha\}) \tag{5}$$

Received April 3, 1973; revision received January 10, 1974.
Index category: Thermal Stresses.

* Structural Mechanics Staff Engineer.

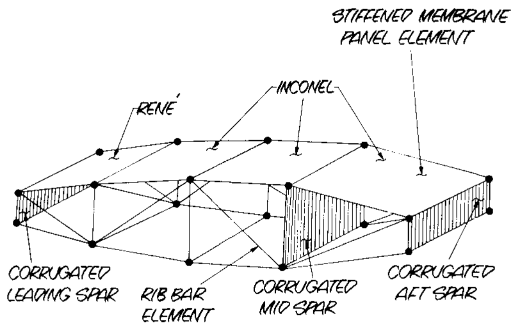


**Fig. 1 Structural idealization between ribs 9 and 10 of H3T shuttle orbiter fin.**

By Eqs. (1, 2, and 4) the dependency of $\{\varepsilon\}$ upon temperature is, to first-order effects†, through $\{P_T\}$, and Eq. (2) shows the $\{P_T\}$ is proportional to an integral involving temperature throughout the entire element. However, Eq. (5) indicates that $\{\sigma\}$ is dependent not only upon $\{P_T\}$ but upon the local temperature as well.

Thus, one can expect a greater accuracy in the numerical calculation of strain than stress in thermal mechanical problems, since some of the errors or approximations in $T$ are self-cancelling in the integrals which determine strain. Alternately stated, local strains are less sensitive to thermal gradients than are their corresponding stresses. Consequently, it is proposed that the strain results for a coarser grid size be coupled with temperature data for a more refined mesh and the stress-strain temperature equations to yield stress results with an accuracy level associated with the finer mesh size.

### Example

To demonstrate the usefulness of the proposed approach for calculating thermal stress, consider the finite element results obtained for the vertical tail of the H3T space shuttle orbiter.[2] The structural idealization between ribs 9 and 10 (there are 20 ribs in all) is shown in Fig. 1. The level of detail depicted is entirely consistent with that required if the loading were purely mechanical.

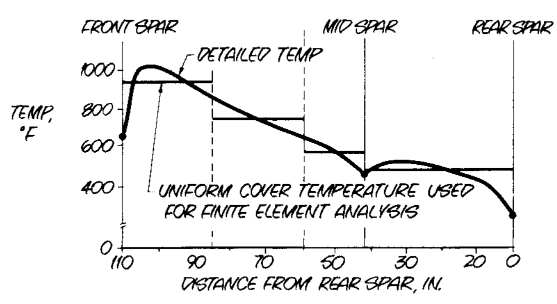


**Fig. 2 Panel temperature between ribs 9 and 10 of H3T shuttle orbiter fin.**

† Higher-order effects are involved as the material properties, and hence the $[D]$ and $\{\alpha\}$ matrices depend upon temperature. However, these dependencies are generally of less importance within a given element than the temperature variation itself.

At 300 sec after a simulated Earth re-entry, the cover panels on opposite sides and between ribs 9 and 10 are heated symmetrically to the temperatures depicted in Fig. 2. As with many of the major general-purpose finite element programs employed, such as those described in Refs. 3–5, each element may be assigned only a single temperature and yields only an average thermal stress for the entire panel. However, as will be shown for the subject problem, average stress results for the associated thermal gradients are entirely unacceptable.

The finite element spanwise strains computed at the panel edges and midpoints of the section in question, compared with those obtained using classical beam theory (plane sections remain plane) were found to be in excellent agreement. Use of beam theory, for comparison purposes, appears justified for the cross section in question, as it is located reasonably outboard from the fin's root attachment section.

The corresponding finite element average membrane stresses calculated by the method presented in Ref. 6, when compared to their beam-theory counterparts, are found to mask critical stress variations within a given element (see Fig. 3). It should be noted that Webber,[7] working with uniform and variable temperature plate membrane elements, came to a similar conclusion. He found that deflections caused by heating were "acceptable" but "special attention" was necessary for "interpolation of the stresses, which could deviate considerably from the true stress state." As may be seen, however, the corresponding stresses computed by the proposed modified approach, also shown in Fig. 3, agree quite well with the beam-theory-based results. The thermal stresses in the chordwise direction are negligible because the cover corrugations permit free thermal straining.

While it may be argued that the uniform temperature elements used in this example are too "primitive," the point is made that the proposed method intrinsically yields more accurate stresses than the conventional approach, regardless of how sophisticated the element employed is.

#### Conclusions and Recommendations

It has been shown by means of a specific example and through general reasoning [Eqs: (1–5)], that the strains which occur as a result of structural heating are less sensitive to local temperature variations than their corresponding thermally induced stresses. Based upon this observation, an improvement over the conventional method for determining thermal stresses employing finite element procedures has been suggested. The technique proposed is equally applicable to a variety of approximate numerical procedures based upon a displacement formulation such as finite differences and other Ritz-type methods. However, the development and example treated have been based upon the finite element method, which is in popular usage because of its flexibility and efficiency in treating complex engineering structures.

Unfortunately, as of this writing, general structural analysis programs such as those described in Refs. 3–5 and 8 do not explicitly output corner or average strains. Thus, implementation of the proposed approach is, at best, awkward. Therefore, it is recommended that such programs be extended to provide a strain print-out option to facilitate the calculation of more accurate thermal stresses.

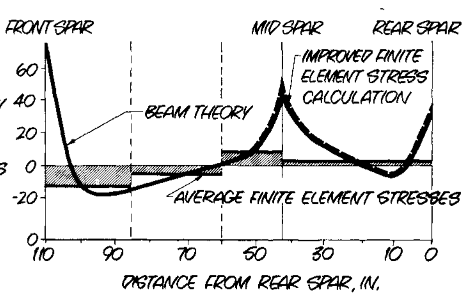


**Fig. 3 Improved finite element fin cover stresses between ribs 9 and 10 vs beam theory and average finite element stresses.**

#### References

[1] Przemieniecki, J. S., *Theory of Matrix Structural Analysis*, McGraw-Hill, New York, 1968, pp. 62–64.

[2] Ojalvo, I. U., "Thermal Dynamic Modeling Study," CR-2125, Oct. 1972, NASA.

[3] MacNeal, R. H., ed., *The NASTRAN Theoretical Manual*, NASA SP-221 (01), April 1972, p. 3.6-3.

[4] *Grumman IDEAS Manual*, Grumman Aerospace Corp., Bethpage, N.Y., Aug. 1968, Chaps. 7 and 27.

[5] Mallett, R. H. and Jordan, S., "MAGIC: An Automated General Purpose System for Structural Analysis," Contract Report for the Air Force, AFFDL-TR-68-56, Jan. 1969, Bell Aerosystems, Buffalo, N.Y.

[6] *Space Shuttle Orbiter TPS Test Bed Design*, Vol. 1, Contract Rept. 71-206-F-NAS, prepared for NASA under Contract NAS 9-12315, April 30, 1972, Grumman Aerospace Corp., Bethpage, N.Y.

[7] Webber, J. P. H., "Thermo-Elastic Analysis of Rectangular Plates in Plane Stress by the Finite-Element Displacement Method," *Journal of Strain Analysis*, Vol. 2, No. 1, 1967, pp. 43–51.

[8] Meijers, P., "Review of the ASKA Programme," Paper II-1 presented at the ONR Symposium on Numerical and Computer Methods in Structural Mechanics, Sept. 1971, Univ. of Illinois at Urbana, Urbana, Ill.

# Shock Tube for Generating Weak Shock Waves

W. Garen,* R. Synofzik,* and A. Frohn†
*Technische Hochschule Aachen, Germany*

THE purpose of the present Note is to introduce a new pneumatic valve which can be used to replace the diaphragm of a conventional shock tube. This device which can be opened practically independently of the pressure difference between driver gas and test gas is particularly useful for generating weak shock waves.

The principle of this device is shown in Fig. 1. The end of tube A is shut off by a rubber disk R. By increasing the

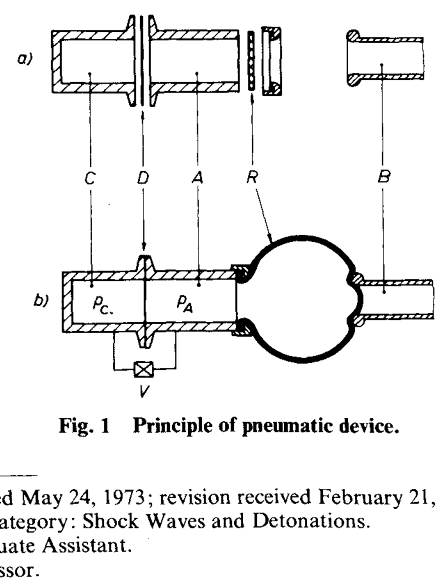


**Fig. 1 Principle of pneumatic device.**

Received May 24, 1973; revision received February 21, 1974.
Index category: Shock Waves and Detonations.

* Graduate Assistant.

† Professor.